E 4400 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2012

Fifth Semester

Core Course—FUZZY MATHEMATICS

(For Model-I B.Sc. Mathematics)

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer **all** *questions.*

Each bunch of **four** *questions has weight* 1.

I. 1. Define an interval-valued fuzzy set.

2. Define level set of a fuzzy set.

3. Write the standard intersection $A \cap B$ of the fuzzy sets.

$A = \cdot 4/v + \cdot 2/w + \cdot 5/x + \cdot 4/y$

$B = \cdot 6/v + \cdot 1/w + \cdot 8/x + \cdot 3/y$

4. Find the α-cut where $\alpha = \cdot 2$ of the fuzzy set $A = \cdot 6/x_1 + 1/x_2 + \cdot 1/x_3 + \cdot 5/x_4$.

II. 5. State the axiomatic skeleton for fuzzy complements.

6. Define Yager class of fuzzy complements.

7. What is the increasing generator of the standard fuzzy complement.

8. Define the pseudo-inverse of a decreasing generator f.

III. 9. Is every convex fuzzy set a fuzzy number ? Justify.

10. Is every ordinary closed interval a fuzzy number ? Justify.

11. Calculate $[4, 10] \cdot [1, 2]$ where $\cdot$ denotes the arithmetic operation of multiplication on closed intervals.

12. If A and B are fuzzy numbers define $A * B$.

IV. 13. Give the canonical form of a unconditional and unqualified fuzzy proposition.

14. Give an example of a truth-qualified proposition.

15. Define Lukasiewicz fuzzy implication.

16. Write the schema of generalized *modus ponens*.

$(4 \times 1 = 4)$

Part B (Short Answer Questions)

Answer any **five** *questions.*

Each question has weight 1.

17. Define the degree of subsethood $S(A, B)$ of a pair of fuzzy sets A and B and show that $0 \leq S(A, B) \leq 1$.

Turn over

18. Prove that a fuzzy set A on $\mathbb{R}$ is convex if and only if $A(\lambda x_1 + (1-\lambda)x_2) \geq \min[A(x_1), A(x_2)]$ for all $x_1, x_2 \in \mathbb{R}$ and all $\lambda \in [0,1]$.

19. If C is a continuous complement, then prove that C has a unique equilibrium.

20. State the axiomatic skeleton for fuzzy intersection/*t*-norms.

21. Define a fuzzy number.

22. Show that X = B – A is not the solution of the fuzzy equation A + X = B.

23. Distinguish between L_∞ and L_1.

24. Consider the proposition "if x is A then y is B", where $A = .5/x_1 + 1/x_2 + .6/x_3$ and $B = 1/y_1 + .4/y_2$. Find the fuzzy set R, given by $R(x, y) = I[A(x), A(y)]$, where I is the Lukasiewicz implication.

(5 × 1 = 5)

Part C (Short Essay Questions)

Answer any **four** *questions.*
Each question has weight 2.

25. State the first decomposition theorem of fuzzy sets (no proof). Illustrate the theorem for the fuzzy set A given by :

$$A(x) = \begin{cases} x-1, & x \in [1,2] \\ 3-x, & x \in [2,3] \\ 0, & \text{otherwise} \end{cases}.$$

26. Let $C:[0,1] \to [0,1]$ be monotonic decreasing function which is innovative. Prove that C is a continuous bijecive function satisfying boundary conditions.

27. Let i_w denote the class of Yager *t*-norms defined by

$$i_w(a,b) = 1 - \min\left(1, \left[(1-a)^w + (1-b)^w\right]^{1/w}\right), w > 0.$$

Prove that $i_{min}(a,b) \leq i_w(a,b) \leq \min(a,b)$.

28. If A, B, C are closed intervals, prove that :

(i) $A \cdot (B + C) \leq A \cdot B + A \cdot C$.

(ii) If $b \cdot c \geq 0$ for every $b \in B$, $c \in C$, then $A \cdot (B + C) = A \cdot B + A \cdot C$.

29. Explain unconditional and qualified fuzzy propositions.

30. Let $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2\}$, $Z = \{z_1, z_2\}$ and $A = \cdot 5/x_1 + 1/x_2 + \cdot 6/x_3$, $B = 1/y_1 + \cdot 4/y_2$, $C = \cdot 2/z_1 + 1/z_2$. Let $I(a, b) = \begin{cases} 1 & \text{if } a \leq b \\ b & \text{if } a > b \end{cases}$

Show that the generalized hypothetical syllogism holds in this case.

(4 × 2 = 8)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question has weight 4.

31. Given an involutive fuzzy complement c of an increasing generator g of c, prove that the t-norm and t-conorm generated by g are dual with respect to c. Further if $\langle i, u, c \rangle$ is a dual triple so generated, prove that the fuzzy operation i, u, c satisfy the law of excluded middle.

32. Let A and B fuzzy numbers defined by :

$$A = \begin{cases} 0 & x \leq 3 \text{ and } x > 5 \\ x - 3, & 3 < x \leq 4 \\ 5 - x, & 4 < x \leq 5 \end{cases}$$

$$B = \begin{cases} 0, & x \leq 12 \text{ and } x > 32 \\ (x - 12)/8, & 12 < x \leq 20 \\ (32 - x)/12, & 20 < x \leq 32 \end{cases}$$

Solve the fuzzy equation $A \cdot X = B$.

33. Explain different types of fuzzy propositions.

(2 × 4 = 8)